Theory of wave activity occurring in the AMPTE artificial comet

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One of the main experiments of the Active Magnetospheric Particle Tracer Explorers (AMPTE) [J. Geophys. Res. **91**, 10013 (1986)] satellite mission was the release of neutral barium atoms in the solar wind. The barium atoms ionized by photoionization extremely rapidly forming a dense, expanding, plasma cloud that interrupted the solar wind flow creating diamagnetic cavities. On the upstream side of the cavity a region of compressed plasma and enhanced magnetic field was created as the result of being produced by the slowing down and deflection of the solar wind, and magnetic field line draping. Intense electrostatic and magnetic turbulence was observed by both the IRM [J. Geophys. Res. **91**, 10 013 (1986)] and UKS [J. Geophys. Res. **91**, 1320 (1986)] satellites at the boundary of the diamagnetic cavity, with the most intense waves being detected near the outer boundary of the compressed region. This paper examines how the newly created expanding plasma couples to the solar wind by means of plasma–beam and current-driven instabilities. In particular, it is shown how lower-hybrid and lower-hybrid drift waves are generated by cross-field proton–barium streaming instabilities and cross-field electron currents. The saturation mechanism for these waves is considered to be the modulational instability, this instability can also lead to filamentation and coupling to magnetosonic modes, which are also observed. As the result of modulational instability the $k_{\parallel}$ component increases, which allows the heating and acceleration of electrons that is consistent with the observations.

I. INTRODUCTION

Recent experiments carried out by the Active Magnetospheric Particle Tracer Explorers (AMPTE)[1] involved the release of lithium and barium atoms in the solar wind. In all cases the expanding cloud of atoms was photoionized by solar ultraviolet radiation, producing an obstacle to the solar wind flow and the formation of diamagnetic cavities. One of the main objectives of the experiments was to understand how the solar wind, in the absence of collisions, interacts with newly created ions. This problem is relevant to a number of astrophysical situations, such as the pickup of cometary ions,[2] the study of planetary ion exospheres interacting with the solar wind,[3] and the pickup of helium and oxygen ions of interstellar or planetary origin.[4] Since both the released plasma and solar wind plasma are collisionless any interaction between them must depend on wave–particle interactions producing the momentum coupling.

The AMPTE releases provide, at the present time, the best detailed data set for the study of artificially created plasma comets and, in general, for the investigation of plasma turbulence created by releases and in flows in the solar wind. Using the results makes it possible to try to formulate a physically self-consistent picture of the whole event including, especially, the different turbulent modes excited and the creation of accelerated electrons. At the present time some of the important parts of this picture have been considered, such as the possible excitation mechanisms for lower-hybrid waves[5,6] and the subsequent electron acceleration by them.[5,7] However, many aspects of the interaction are still waiting to be understood even qualitatively, such as the observed level of ion-sound turbulence inside the cavity,[8] the current filamentary structure, the slowing down and heating of the solar wind protons, and the heating of the barium ions.[9] We believe that for different parts of the observed phenomena several explanations can be given. We would like, though, to construct a general picture, which will be able to give qualitative answers to the most important observations, if not all, and find out the relationship between the different phenomena observed, such as the nature and level of fluctuations observed both inside the diamagnetic cavity, at the boundary between the cavity and the shocklike structure, and within the shocklike structure. It seems very likely that all these processes are related, making it less likely that a number of different processes are responsible. In the present paper we propose one main self-consistent picture of the turbulence excited by the AMPTE release. Such a model will help in the understanding of future possible experiments. A

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similar approach can be used for the bow-shock, aurora, and other precipitation phenomena. In considering a self-consistent approach we bear in mind the necessity of estimating the most probable nonlinear processes going on in a consistent manner.

One important point should be mentioned before we start the discussion, namely that one could not expect from the point of view of the present understanding of nonlinear turbulent process, that the broad spectra observed are strongly related to the frequencies of linear modes or to the wavelengths of the fastest growing modes one can only obtain best estimates for such values. The opposite should be expected, namely as a result of nonlinear processes the turbulence should have a broad spectra with a maximum intensity, not at the wavelengths of the maximum growth rates, or at the frequency corresponding to the maximum growth rate, most of the observed features will be the result of turbulent nonlinear processes, with a few features being describable by linear theory.

II. MAJOR OBSERVATIONAL DATA ON THE AMPTE BARIUM RELEASE

Initially before the barium release the solar wind parameters were electron density $n_e \simeq 2\ \text{cm}^{-3}$, velocity of solar wind $v_{sw} \simeq 550$ km/sec, electron temperature and magnetic field $T_e \approx 20$ eV, $H \approx 10^{-4}$ G or 10 nT. For these parameters the Debye length is 20 m and the lower-hybrid resonance frequency $\omega_{LH} \approx (\omega_{Hi}\omega_{He})^{1/2} \simeq 20\ \text{sec}^{-1}$. The interaction of released ions with the solar wind produces a diamagnetic cavity, of the order of 70 km in radius, surrounded by a pile-up region of compressed plasma and high magnetic field strength of the order of 100 nT.[10] The scale size of the magnetic field structure is of the order of 1–10 km or less, with a gradual increase on a scale length of up to 200 km. The magnetic structure surrounding the cavity is influenced by the solar wind and is very filamentary.[10] The size of the filamentary structures could be less than 1 km or even 100 m. The cavity is surrounded by a shocklike structure in the upstream region where there exists a large magnetic field structure of the order of 60–100 nT. The lower-hybrid frequency for barium ions in this region is of the order of 30–100 Hz, while the proton plasma frequency in this region is of the order of kilohertz, which correspond to the range of frequencies of intense electrostatic noise observed mainly in the region surrounding the cavity or between the cavity and the upstream solar wind. A plot of number density magnetic field and plasma wave electric field data obtained during the AMPTE barium release (taken from Ref. 10) is shown in Fig. 1. The plasma density of the barium ions (singly ionized barium, i.e., the density of cold electrons appearing in the cavity is of the order of the density of barium ions) varies from 10^5–$10^4\ \text{cm}^{-3}$ in the center down to almost the solar wind density at the upstream side of the cavity. Inside the cavity ion sound turbulence is observed and maintained for a period of observation (10 min), although the Landau damping of ion sound waves is of the order of or less than one-tenth of a second, this demonstrates a necessity for a source driving the ion sound turbulence. The wave turbulence in the cavity is at a very low level, in comparison with the compressed region, as shown in Fig. 1. The level of the turbulent fields observed will be given below in connection with theoretical estimates.

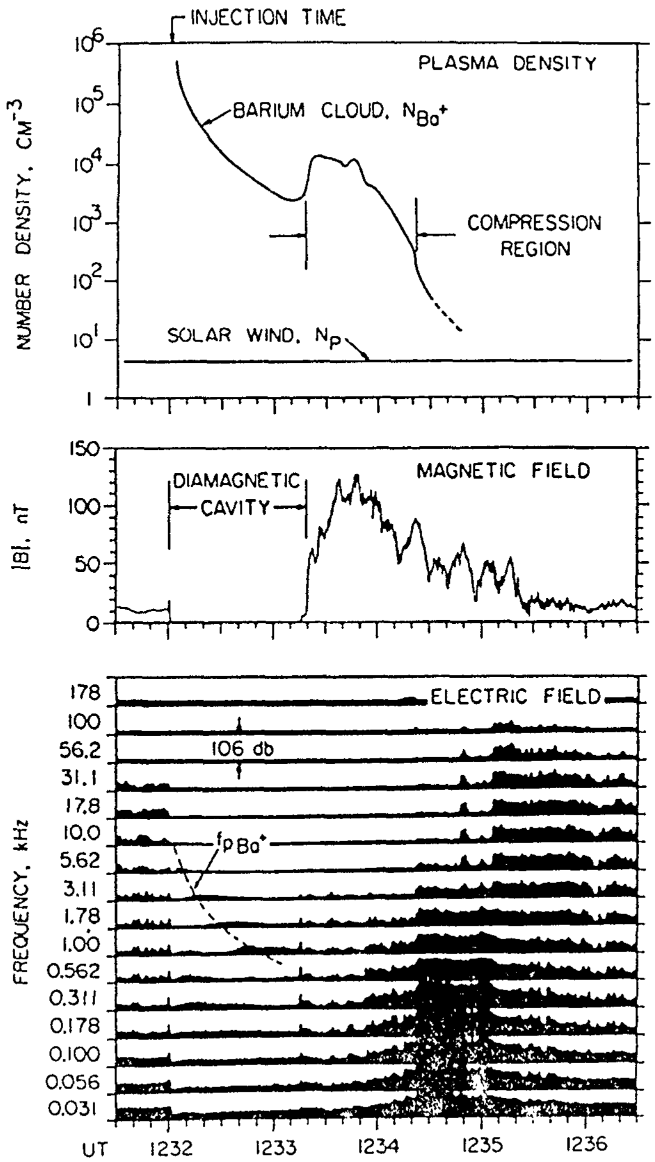

FIG. 1. Magnetic field profile and wave electric field spectrum taken from Ref. 10. The data were obtained during the AMPTE barium release in the solar wind. Note the existence of intense oscillations in the frequency range 30–1 Hz at the outer part of the plasma cloud.

In the region of the large field compression at the front of the cloud and, more importantly, in the whole of the turbulent region between the shock and the cavity electron energization (acceleration) is observed. Electrons with energies greater than 100 eV are created. Solar wind ions are slowed down and deflected in this region. The most intense wave activity is also observed in the region of enhanced compression. The slowing down of the solar wind to 270 km/sec also occurs in this region, corresponding to a flow energy of 0.4 keV. The initial flow energy is about 1.6 keV. This means that the solar wind loses about 70% of its flow energy and it is heated up to 0.4 keV at the rear of the wave activity. The barium ions can be considered practically as not moving. The most intense waves appear at a frequency of 30 Hz at

1234:30 the time of maximum magnetic field activity encountered.

III. A GENERAL PHYSICAL PICTURE OF THE WAVE ACTIVITY

It is obvious that in the strongly turbulent region the magnetic fields are important and the interaction of solar wind protons with barium ions should be treated with the magnetic field taken into account, particularly since some of the wave activity observed is in the range of lower-hybrid frequencies i.e., some of the modes excited require the presence of the magnetic field (in this sense the analysis of reference[8] is not quite adequate to describe the whole problem). Instead of a reflected ion beam, as at the Earth's bow shock, the two-beam situation in the barium release appears as a result of the interaction of the solar wind protons with the almost stationary barium ions.

The ion–ion beam interaction should excite two types of waves, a long wavelength mode $k \simeq \omega_{pe}/c$ in the vicinity of the lower-hybrid frequency and the shorter wavelength $k \sim \omega_{pe}/v_{Te}$ ion-acoustic mode. The most intensive wave activity is to be connected with the ion-acoustic mode, the frequency of this mode, as measured by the spacecraft, varies over a wide range from tens of hertz up to kilohertz. Simple estimates based on quasilinear theory give for the wave electric field spectral density the value 10–15 $\text{mV/m}\sqrt{\text{Hz}}$, the growth rate increases with n_B—the density of the barium beam—and as a consequence the instability mechanism under consideration is important in the whole turbulent zone between the shock and the cavity. The effective excitation of the lower-hybrid mode is possible only in the case of oblique propagation with respect to the magnetic field. The excitation takes place in the outer part of the turbulent zone, where $n_B \leqslant n_0(m_B/m_p)$, $m_{B,P}$ is barium; proton mass. For larger n_B the growth rate of the lower-hybrid mode γ decreases as $n_B^{-1/3}$, thus making the excitation of this mode at $n_B \sim 10^4–10^5$ impossible since the time needed for excitation $10/\gamma$ becomes larger than the convective time $L/u(L \simeq 300$ km—the width of the turbulent zone, $u \simeq 270$ km/sec the speed of the solar wind slowed down by the cavity).

Another type of instability to be considered is the lower-hybrid drift instability induced by the currents. The largest currents are the diamagnetic currents $[\mathbf{J}_D = -\nabla(nT)\times\mathbf{H}/H^2]$ existing at the boundary of the cavity. The growth rate of this instability is limited only by the time scale during which the current flows and has therefore less rigid conditions of applicability than the proton–barium beam instability, i.e., it can also occupy the whole turbulent region. It also exists in the region of high ion (barium) density and is not suppressed with increasing density. But there does exist a threshold for the drift velocity of the current ·arrying electrons to drive this instability. This threshold is ·t large—the electron drift velocity necessary could be ·h less than the ion thermal velocity. The saturated level ·ctrostatic fluctuations excited by both types of lower- instabilities are, as estimations show, determined by ·ulational instability, which is somewhat different from the usual modulational instability, because of the kinetic pressure of the barium ion flow in the solar wind frame. In this case the nonlinear coupling between the lower hybrid waves occurs through the low-frequency magnetic field perturbations. Slowly varying magnetic field structures having wave numbers $k \ll \omega_{pe}/c$, which corresponds to characteristic space scales of about 1 km or less, should develop. In places of low magnetic field the frequency of the lower-hybrid mode is decreasing, hence these places serve as potential wells in which the wave quanta of the lower-hybrid mode can be trapped. This trapping results in the modulation of the lower-hybrid wave intensity that further increases modulation of the magnetic field under the action of the ponderomotive force of the wave.

The presence of fine scale low-frequency magnetic field structures follows from observations obtained by Lühr *et al.*[11] The creation of these magnetic field structures resulting from the modulational instability of lower-hybrid waves will result, according to Ampère's law, in currents with $\mathbf{J} = (c/4\pi)\nabla\times\mathbf{H}$ sufficient to excite the lower-hybrid drift modes. It is important that the development of the modulational instability lead to the appearance of waves with $k_\parallel \simeq (m_e/m_B)^{1/2}k_{\text{LH}}$, which can be absorbed by the tail of the electron distribution. In the initial stage of the instability one could divide the turbulent region between the solar wind and the inner boundary of the cavity into two regions. In the outer part the instability of lower-hybrid waves driven by the two beam instability (solar wind proton plus the barium ions) develops. In the inner region the lower-hybrid drift waves are excited by the diamagnetic currents flowing as a consequence of Ampère's law because of the fine scale magnetic field structures.

Finally the stage will be reached when in the whole turbulent region, including that part where initially only the beam-driven lower-hybrid instability develops, the lower-hybrid drift mode will also exist. These lower-hybrid waves of both types are absorbed by electrons resulting from Landau damping and energize them. The time of energization should be less than the confinement time of electrons in the turbulent region. In this context it is important that lower-hybrid drift modes exist in the whole turbulent region. Otherwise for the usual lower-hybrid instability, the outer part of the turbulent region, where it is effectively excited, seems to be insufficient in size to accelerate the electrons. The fastest electrons are lost thus increasing the electrostatic potential of the turbulent region up to their escape energy (say 100 eV). This dc electric field cannot produce a large current because of the large value of the effective collision frequency inside the turbulent region. These processes can also explain the heating of protons observed, first of all via the beam–plasma-driven ion-acoustic instability. Since the ion-acoustic instability considered in this paper is of the negative energy type, in the process of increasing of their amplitudes the waves heat the resonant particles (barium or proton ions) by an amount equal to their energy gain.

The proposed mechanisms are therefore the development of broadband intense electrostatic turbulence as the result of the ion-acoustic instability as well as lower-hybrid turbulence that could also be responsible for fine scale magnetic field structures. Both instabilities are driven by barium beam solar wind plasma interaction. The currents associated with the magnetic field structures could drive another type of lower-hybrid instability—namely lower-hybrid drift instability. This turbulence is finally absorbed by fast electrons, explaining the appearance of high energetic electron tails in the turbulence zone.

The above scenario will be similar to the case of a shock and particle precipitation. Indeed in the case of a bow shock the role of the second beam is played by the protons reflected by the shock, and in the case of precipitation the flux of fast ions in the beam. The lower-hybrid drift waves appear at the final stage and can also explain the filamentary structure of magnetic fluctuations at the shock and perhaps also the filamentary structure of aurora.

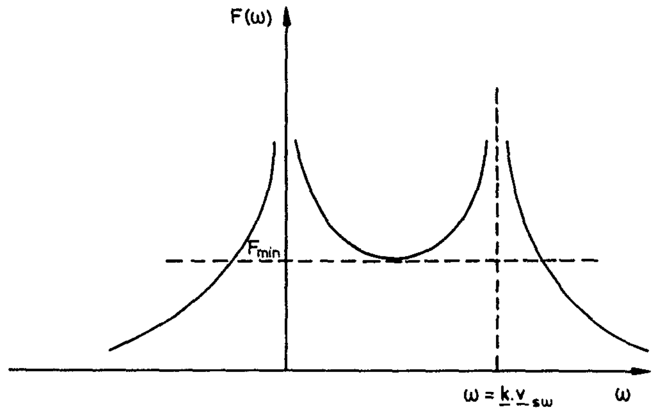

FIG. 2. A graphical representation of $F(\omega)$ given by Eq. (2), as a function of ω from Eq. (1); values of $F_{\min} > 1 + \omega_{pe}^2/k^2c^2$ indicate instability.

IV. LINEAR THEORY OF THE VARIOUS INSTABILITIES

A. Proton–barium beam–plasma instability

We start from the investigation of long wavelength instabilities ($k \sim \omega_{pe}/c$) resulting in the excitation of waves with the frequencies in the vicinity of the lower-hybrid resonance. Electrons are magnetized in such oscillations where $kv_{Te} \ll \omega_{He}$ and $\omega \ll \omega_{He}$. The main plasma is the ionized barium gas; solar wind protons form a cold beam in such a plasma. The dispersion relation for oblique ($k_\parallel \neq 0$) waves excited resulting from the beam–plasma interaction has the form

$$\frac{\omega_{\rm LH}^2}{\omega^2}\frac{m_p}{m_B} + \frac{\omega_{He}^2}{\omega^2}\frac{k_\parallel^2}{k^2+\omega_{pe}^2/c^2} + \frac{\omega_{\rm LH}^2}{(\omega - \mathbf{k}\cdot\mathbf{v}_{\rm sw})^2}\frac{n_p}{n_B} = 1 + \frac{\omega_{pe}^2}{k^2c^2}. \qquad (1)$$

Here $\omega_{\rm LH} = \sqrt{\omega_{He}\omega_{Hp}}$ is the lower-hybrid frequency in a dense ($\omega_{pe} \gg \omega_{He}$) solar wind plasma, and n_B, n_p, m_B, and m_p are the densities and masses for barium ions and protons, respectively. For the two species case (protons flowing through electrons) this equation was obtained and analyzed by McBride *et al.*,[12] and for the plasma and magnetic field inhomogeneities taken into account by Hsia *et al.*[13] The effect of inhomogeneity leading to the lower-hybrid drift instability will be analyzed in the next section. Here we will restrict ourselves by analyzing the homogeneous case and our aim will be to understand how the presence of a third (barium) component will influence the modified two-stream instability described by Eq. (1). For the interpretation of wave data in the AMPTE releases an equation similar to Eq. (1), but with $k_\parallel = 0$, was used by Papadopoulos *et al.*[6] It was shown in this reference that a release of barium ions forming a beam in the solar wind plasma could lead to the excitation of lower-hybrid waves propagating across the magnetic field. We will show below that for oblique ($k_\parallel \neq 0$) waves, the region where such excitation takes place expands considerably. We consider the region where $n_B \gg n_p$ and hence use the condition of quasineutrality in the form $n_e \simeq n_B$, i.e., the electron density is balanced by the barium ion density. The plot of the left-hand side of Eq. (1), denoted by $F(\omega,k)$ as a function of ω, is shown in Fig. 2. The minimal value of F is given by

$$F_{\min} = (n_p/n_B)[\omega_{\rm LH}^2/(\mathbf{k}\cdot\mathbf{v}_{\rm sw})^2](1+\alpha^{1/3})^3, \qquad (2)$$

where the following notation:

$$\alpha(k,k_\parallel) = \frac{n_B}{n_p}\frac{m_p}{m_B} + \frac{k_\parallel^2}{(k^2+\omega_{pe}^2/c^2)}\frac{m_p}{m_e}\frac{n_B}{n_p}$$

is used.

For instability the dispersion relation Eq. (1) must have at least two complex roots; the condition for this is

$$\mathbf{k}\cdot\mathbf{v}_{\rm sw} < kv_{\rm Ap}(n_p/n_B)[1+\alpha^{1/3}(k,k_\parallel)]^{3/2}. \qquad (3)$$

Here $v_{\rm Ap} = H_0/\sqrt{4\pi n_p m_p}$ is the Alfvén velocity in the solar wind plasma. For transverse magnetic field propagation ($k_\parallel = 0$), $\alpha \ll 1$, corresponding to the situation in the AMPTE barium release. The condition for waves propagating almost along the beam direction, i.e., $\mathbf{k}$ parallel to $\mathbf{v}_{\rm sw}$, is

$$n_B < n_p(v_{\rm Ap}/v_{\rm sw}) \qquad (4)$$

for the case $n_p = n_e$; this coincides with the condition obtained by Hsia *et al.*[13] It follows from this relation that for the conditions of the AMPTE experiment the instability under consideration is possible only in the outer part of the barium cloud, where

$$n_B < (5\text{–}10)n_p.$$

The wave number of the fastest growing mode in the case $\alpha \ll 1$ is equal to

$$k = \sqrt{\frac{\omega_{\rm LH}^2}{v_{\rm sw}^2}\frac{n_p}{n_B} - \frac{\omega_{\rm LH}^2}{v_{\rm Ap}^2}\frac{n_B}{n_p}}. \qquad (5)$$

The frequency and the growth rate of that mode are obtained from the relation

$$\omega = (\alpha/2)^{1/3}[(1+i\sqrt{3})/2]\mathbf{k}\cdot\mathbf{v}_{\rm sw}. \qquad (6)$$

In a more dense part of the cloud $n_B \leqslant n_p(m_B/m_p) \simeq 10^2 n_p$, it follows from Eq. (3) that the instability is possible only for the case where $\alpha \gg 1$ for oblique ($k_\parallel \neq 0$) waves. For the case $n_e = n_p$ the instability of oblique waves was analyzed by Gurnett *et al.*[14] The frequencies of excited waves for the case $\alpha \gg 1$ are obtained from the relation

$$\omega = \frac{\omega_{\rm LH}}{\sqrt{1+\omega_{pe}^2/k^2c^2}}\sqrt{\frac{m_p}{m_B} + \frac{k_\parallel^2}{k^2+\omega_{pe}^2/c^2}\frac{m_p}{m_e}}. \qquad (7)$$

For the waves with $k_\parallel/k \gg \sqrt{m_e/m_B}$, the waves correspond to oblique electrostatic waves with $\omega = \omega_{He} k_\parallel/k$ for $\omega_{pe} \ll kc$, and the whistler mode with $\omega = \omega_{He}(c^2 k_\parallel k/\omega_{pe}^2)$ for $\omega_{pe} \gg kc$. The growth rate of the most unstable mode for $\alpha \gg 1$ is also easily obtained from Eq. (1) and is given by

$$\gamma = (\sqrt{3}/2^{4/3})[\omega/\alpha^{1/3}(k,k_\parallel)]. \quad (8)$$

For example, for the AMPTE conditions where $v_A \simeq 2\times 10^8$ cm/sec ($H_0 \simeq 100$ nT), $v_{sw} \sim 2.7\times 10^7$ cm/sec the excitation of waves in the region $n_B = n_p m_B/m_p$ is possible in the case $\alpha \gtrsim 10^2 (k_\parallel/k \gg \sqrt{m_e/m_p})$, the frequency in this case is $\omega \gg \omega_{LH} \simeq 2.5\times 10^2$ sec^{-1}. For $\omega \simeq \omega_{LH}$, $\alpha \simeq 10^2$ the growth rate obtained from Eq. (8) is $\gamma \sim 40$ sec^{-1}, and the instability growth length $L \sim 10 v_{sw}/\gamma \sim 10^7$ cm is quite sufficient for the excitation of waves in the turbulent zone. In the more dense region a further decrease of γ (γ drops as $n_B^{-1/3}$ for fixed ω) makes it impossible to excite the waves in that frequency range. The similar decrease of γ makes the excitation of waves propagating at large angles θ toward the beam impossible. In this case it follows from Eq. (8) that the instability region is shifted toward larger n_B but the growth rate decreases proportionally to $\cos\theta$, in accordance with Eq. (6). It also follows from the analysis of nonlinear saturation resulting from beam trapping (see below, Sec. IV A) that the amplitudes of the excited waves in the case of small $\cos\theta$ are extremely small, i.e., $(|E|^2 \propto \cos^4\theta)$.

The possibility of ion-acoustic mode excitation in the AMPTE releases was considered by Gurnett *et al.*[14] In this reference (also see Ma *et al.*[15]) a detailed numerical analysis of the appropriate dispersion relation in an unmagnetized plasma was presented. It is the aim of the present publication to clarify the physical picture of the ion-acoustic instability by an analytical analysis of the dispersion relation. In particular, it will be shown that the main mechanism for the growth of the ion-acoustic mode in the region of a dense barium plasma ($n_B \gg n_p$) is the dissipative instability as a result of the negative energy of this mode.

It is more convenient to consider the excitation of short wavelength ion-acoustic modes in the solar wind reference frame, in which the barium ions form a cold beam. We will start from the usual dispersion relation for the electrostatic oscillations in a magnetized plasma,[16] which can be written as

$$1 + \frac{4\pi}{k^2}\sum_\alpha \frac{e_\alpha^2}{m_\alpha}\int d\mathbf{v}\left(\frac{k_\perp}{\lambda_\alpha}\frac{\partial f_0^\alpha}{\partial v_\perp}\sum_{n=-\infty}^{\infty}\frac{n I_n^2(\lambda_\alpha)}{\omega - k_\parallel v_z - n\omega_{H\alpha}} + k_\parallel \frac{\partial f_0^\alpha}{\partial v_z}\sum_{n=-\infty}^{\infty}\frac{I_n^2(\lambda_\alpha)}{\omega - k_\parallel v_z - n\omega_{H\alpha}}\right) - \frac{\omega_{pb}^2}{(\omega + \mathbf{k}\cdot\mathbf{v}_{sw})^2} = 0, \quad (9)$$

're I_n is the modified Bessel function and $k_\perp v_\perp/\omega_{H\alpha}$, the summation over α is for all plasma nents (electrons + solar wind protons). The last the input from the cold barium beam and $\pi e^2 n_b/m_B$. We have assumed that the growth rate ibility is such that $\gamma \gg \omega_{HB}$, the barium gyrofre- he beam is considered to be unmagnetized. Using f the plasma particles distribution function it is possible to rewrite the integral over $\mathbf{v}$ in the dispersion relation in the form

$$I = 2\pi \int dv_z f_0(0,v_z) + \omega \int \frac{d\mathbf{v}}{v_\perp}\frac{\partial f_0}{\partial v_\perp} \times \sum_{n=-\infty}^{\infty}\frac{I_n^2(\lambda)}{\omega - k_\parallel v_z - n\omega_H}.$$

We also assume that the distribution function for plasma particles are Maxwellian and we shall perform the integration over $v_z, v_\perp$ and summation over n with the help of the tabulated results in Ref. 17, we finally obtain

$$I = \frac{n_0 m}{T}\left[1 + i\omega \exp\left(-\frac{k_\perp^2 T}{m\omega_H^2}\right)\int_0^\infty d\xi \times \exp\left(i\omega\xi - \frac{k_\parallel^2 T}{2m}\xi^2 + \frac{k_\perp^2 T}{m\omega_H^2}\cos\omega_H\xi\right)\right]. \quad (10)$$

In a weak magnetic field $\omega_H \ll k_\parallel\sqrt{T/m}$ the integral ξ could be reduced to the well-known probability integral and we can obtain the following dispersion relation:

$$1 + \sum_\alpha \frac{1}{k^2 d_\alpha^2}\left\{1 + \sqrt{\frac{\pi}{2}}\, i\frac{\omega}{k}\sqrt{\frac{m_\alpha}{T_\alpha}}\exp\left(\frac{-m_\alpha\omega^2}{2T_\alpha k^2}\right) \times\left[1 + \Phi\left(i\frac{\omega}{k}\sqrt{\frac{m_\alpha}{2T_\alpha}}\right)\right]\right\} - \frac{\omega_{pb}^2}{(\omega + \mathbf{k}\cdot\mathbf{v}_{sw})^2} = 0, \quad (11)$$

where d_α is the Debye length and $\Phi(x) = (2/\sqrt{\pi})\int_0^x e^{-t^2}dt$ is the probability integral. This is the dispersion relation used in Ref. 14 for the investigation of the ion-acoustic instability. The conditions for its applicability are $\omega_{H\alpha} \ll k_\parallel v_{T\alpha}$ for plasma particles and $(\omega + \mathbf{k}\cdot\mathbf{v}_{sw}) \gg k_\parallel v_{T_B}$, $\gamma \gg \omega_{H_B}$ for the barium beam. First of all we will consider the case of cold solar wind protons, i.e., $\omega \gg k_\parallel v_{Tp}$, then the dispersion relation has the following form:

$$1 + \frac{1}{k^2 d_e^2} - \frac{\omega_{pp}^2}{\omega^2} - \frac{n_B}{n_p}\frac{m_p}{m_B}\frac{\omega_{pp}^2}{(\omega + \mathbf{k}\cdot\mathbf{v}_{sw})^2} = 0, \quad (12)$$

where $d_e = \sqrt{T_e/4\pi e^2 n_e}$ is the electron Debye length and ω_{pp} is the proton plasma frequency. Considering the situation where $\alpha_0 \equiv (n_B/n_p)(m_p/m_B) \ll 1$, then for this case the frequency of the excited wave is the usual frequency of the ion-acoustic mode, namely,

$$\omega_s = \omega_{pp} k d_e/\sqrt{1 + k^2 d_e^2}. \quad (13)$$

The Čerenkov resonance condition with a beam becomes $\omega_0 \simeq -\mathbf{k}\cdot\mathbf{v}_{sw}$ and it is easy to see that the excited waves propagate almost perpendicular to the ion beam with $\mathbf{k}\cdot\mathbf{v}_{sw}/|k v_{sw}| \leqslant (1/v_{sw})\sqrt{T_e/m_p(n_p/n_B)} \approx 10^{-1}$–$10^{-2}$. Substituting into the dispersion relation $\omega = -\mathbf{k}\cdot\mathbf{v}_{sw} + \delta\omega$, and using $|\delta\omega| \ll \omega_0$ we have from Eq. (12):

$$\delta\omega = \omega_0(\alpha_0/2)^{1/3}[(1 + i\sqrt{3})/2]. \quad (14)$$

The condition of applicability of Eq. (12) is $\omega \gg k v_{Tp}$, which, with the help of Eq. (13), can be written in a form $T_p/T_e \ll n_p/n_B$, and it can only be fulfilled in the outer part of the cloud, where the density of barium ions does not exceed very significantly the solar wind density. More importantly is the opposite case with $\omega \ll kv_{Tp}$ when the beam-driven instability is of a dissipative nature. In that case the dominant dissipation is due to the solar wind protons and the dispersion relation has the form

$$1+\frac{1}{k^2d_e^2}+\frac{1}{k^2d_p^2}+i\sqrt{\frac{\pi}{2}}\frac{\omega}{kv_{Tp}}\frac{1}{k^2d_p^2} - \alpha_0\frac{\omega_{pp}^2}{(\omega+\mathbf{k}\cdot\mathbf{v}_{sw})^2}=0, \quad (15)$$

the solution of which, for $\alpha_0 \ll 1$, is the following:

$$\omega+\mathbf{k}\cdot\mathbf{v}_{sw} = -\frac{\alpha_0^{1/2}\omega_{pp}kd_*}{\sqrt{1+k^2d_*^2}} \times\left(1-i\sqrt{\frac{\pi}{8}}\frac{\omega}{kv_{Tp}}\frac{d_*^2/d_p^2}{1+k^2d_*^2}\right), \quad (16)$$

where we have used the notation

$$\frac{1}{d_*^2}=\frac{1}{d_p^2}+\frac{1}{d_e^2}=4\pi e^2\left(\frac{n_p}{T_p}+\frac{n_e}{T_e}\right).$$

The condition for the instability developing,

$$|\mathbf{k}\cdot\mathbf{v}_{sw}|>\alpha_0^{1/2}\omega_{pp}kd_*/\sqrt{1+k^2d_*^2}, \quad (17)$$

corresponds to a negative energy wave, as is usual in the case of dissipative-type instabilities. It follows from Eq. (16) that the growth rate of the instability increases with an increase in the barium ion density ($\gamma \sim n_B^{1/2}$). The solution of the dispersion relation corresponding to the very dense barium plasma region $\alpha_0 \gg 1$ is

$$\omega+\mathbf{k}\cdot\mathbf{v}_{sw}=\frac{(1+i)}{\sqrt{2}}\sqrt{\frac{2}{\pi}\frac{kv_{Tp}}{|\mathbf{k}\cdot\mathbf{v}_{sw}|}}\,\alpha_0^{1/2}\omega_{pp}, \quad (18)$$

which yields a growth rate $\gamma \propto n_B^{1/2}$, as in the previous case. Hence, contrary to the lower-hybrid mode, the excitation of the ion-acoustic mode takes place throughout the whole cloud, where the solar wind is present. The maximum growth rate corresponds to short wavelength ion-acoustic oscillations with $kd_* \simeq 1$. The frequency of oscillations, as measured by the spacecraft given by $\omega+\mathbf{k}\cdot\mathbf{v}_{sw}$, is of the order of the barium ion plasma frequency ω_{pB}, which is of the order of kilohertz close to the compressed region.

B. Current-driven lower-hybrid mode instability

As mentioned before the large gradients of magnetic fields in the turbulent zone of the cloud result in diamagnetic currents flowing as a consequence of Ampère's law. The lower-hybrid drift instability driven by plasma and magnetic field inhomogeneities was considered previously by Hsia *et al.*[13] and Davidson *et al.*,[18] and the lower-hybrid drift instability initiated by an electron current was considered by Sotnikov *et al.*[19] In all these cases the instability is due to the negative energy of the lower-hybrid mode and develops as a result of wave damping on thermal protons, we call this instability a kinetic one. In a three-component plasma created by the AMPTE releases, another type of instability, in which the dissipation is due to the cold barium beam, develops, namely a hydrodynamical lower-hybrid drift instability described in the solar wind frame by the following dispersion relation:

$$\frac{\omega_{pe}^2}{\omega_{He}^2}\left(1+\frac{\omega_{pe}^2}{k^2c^2}\right)-\frac{\omega_{pB}^2}{(\omega+\mathbf{k}\cdot\mathbf{v}_{sw})^2}-\frac{\omega_{pe}^2\mathbf{k}_y\kappa}{\omega_{He}(\omega-k_yu_{ey})k^2} +\frac{1}{k^2d_p^2}=0. \quad (19)$$

Here we have assumed that the mode under consideration is polarized in the plane perpendicular to the magnetic field with $k_\parallel=0$. The inhomogeneity of the density and the magnetic field profiles perpendicular to the magnetic field is characterized by the inhomogeneity scale length κ^{-1}, defined by

$$\kappa=\frac{1}{n_0}\frac{dn_0}{dx}-\frac{1}{H}\frac{dH}{dx}.$$

In obtaining Eq. (19) we have assumed that in the expanding plasma cloud there is no balance between gas kinetic and magnetic pressures as the result of the sufficiently fast expansion compared to the time scale of the barium ion gyroperiod ω_{HB}^{-1}. In this case the inhomogeneity of the magnetic field profile is supported by an electron current with velocity

$$u_{ey}=\frac{c}{4\pi en_0}\frac{dH}{dx},$$

where n_0 in the definition of κ is the electron density; the geometry of the background configuration is shown in Fig. 3. It should also be noted that for the instability of a stationary plasma configuration with an inhomogeneous magnetic field and dispersion equation similar to Eq. (19) (without using the approximation of hot ions) was obtained by Davidson *et al.*[18] In that case $u_{ey}=-c(E_0/H_0)$ in Eq. (19) is the equilibrium $\mathbf{E}\times\mathbf{H}$ velocity, the gradient of the magnetic field is supported by the diamagnetic electron drift velocity $v_{eyd}=-(T_e/m_e\omega_{He})(d/dx)\ln n_0$ and

$$\frac{dH}{dx}=\frac{4\pi en_0}{c}(v_{ey}-v_{iy})$$

(the ion contribution to the diamagnetic current is negligible in this case because of the low temperature of barium ions and the low density of protons). In the AMPTE release such

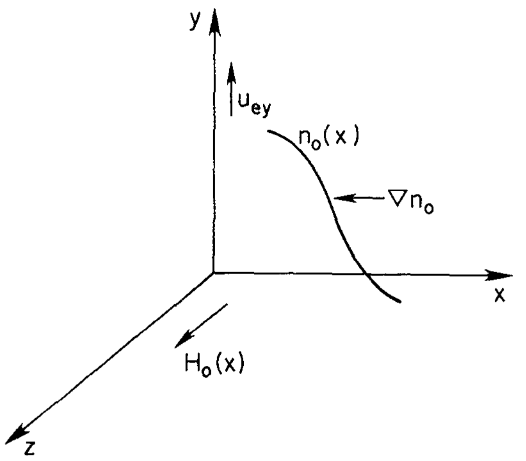

FIG. 3. A schematic representation for the density inhomogeneity and magnetic field configuration for Eq. (19).

a type of configuration could only be realized in the inner part of the plasma cloud closer to the diamagnetic cavity.

In the case of sufficiently small κ the lower-hybrid drift instability is of a resonant nature, where

$$\omega - k_y u_{ey} = \epsilon, \quad |\epsilon| \ll \omega.$$

At the same time $|\epsilon|$ and the gradient scale length κ must be sufficiently large in order to have a hydrodynamical instability [see condition (22)]. In this case we have from Eq. (19) the following equation for the frequency of the unstable mode:

$$\omega = -\mathbf{k}\cdot\mathbf{v}_{\rm sw} + \omega_{\rm sp} \equiv -\mathbf{k}\cdot\mathbf{v}_{\rm sw} + \omega_{{\rm LH}B} k r^* \sqrt{n_B/n_e}/\sqrt{1+\beta+k^2 r^{*2}} \quad (20)$$

and its growth rate $\gamma = {\rm Im}\,\epsilon$, where

$$\epsilon^2 = \frac{n_p}{n_B}\frac{m_B}{m_p}\frac{k_y\kappa}{k^2}\frac{\omega_{\rm sp}^3}{\omega_{Hp}}. \quad (21)$$

Here $\omega_{\rm sp}$ is the frequency measured by the spacecraft, and the following definitions for r^* and β have been used:

$$r^* = \sqrt{\frac{T_p}{m_e}}\frac{1}{\omega_{He}}\sqrt{\frac{n_e}{n_p}}, \quad \beta = \frac{T_p}{m_e v_{Ae}^2}\frac{n_e}{n_p},$$

where $v_{Ae} = c\omega_{He}/\omega_{pe}$ is the electron Alfvén velocity, $\omega_{{\rm LH}B} \simeq \sqrt{m_p/m_B}\,\omega_{\rm LH}$ is the lower-hybrid frequency for barium ions. The typical value of the frequency in the spacecraft reference frame does not exceed $(1/2\pi)\omega_{{\rm LH}B}$ or (10–15)Hz.

The mechanism of the instability is quite a simple one. The necessary condition for the instability is $\kappa < 0$, giving rise to a negative energy drift mode and any dissipation mechanism connected with the absorption of energy by barium ions or by solar wind protons, leads to an instability. In the case of the hydrodynamical instability the absorption of energy by cold barium ions is important. In obtaining the growth rate [Eq. (21)] we have neglected the input from the wave damping resulting from the solar wind protons; the condition for this is

$$\frac{|\epsilon|}{\omega} \gg \frac{n_p}{n_B}\frac{m_B}{m_p}\left(\frac{\omega_{\rm sp}}{k v_{Tp}}\right)^3, \quad (22)$$

which is more easily fulfilled in the region of dense barium plasma. The growth rate of the hydrodynamical lower-hybrid drift instability for $kr^* < 1$ (increasing with n_B) is $\gamma \sim n_B^{1/4}$. For example, let us consider the region of dense plasma and "piled-up" magnetic field $n_e \simeq n_B \simeq 10^4\ {\rm cm}^{-3}$, $H \simeq 1.5\times 10^{-3}$ G. In that region $\omega_{pe}/2\pi \simeq \frac{1}{2}\times 10^6$ Hz, $\omega_{He}/2\pi \simeq 4\times 10^2$ Hz the electron current velocity $u_{ey} \simeq 10^6$ cm/sec, here we also used $\Delta x \simeq 3$ km for the characteristic space scale of the inhomogeneity in the "piled-up"region. The growth rate obtained from Eq. (21) is $\gamma \simeq 5\ {\rm sec}^{-1}$ and the typical growth length of the oscillations is $L \simeq 10 v_{\rm sw}/\gamma \sim 10^7$ cm, which is quite sufficient for the development of the instability. The experimental results[6,8] shows the presence of intense oscillations down to the frequency of the lower-hybrid barium resonance in the region containing the whole turbulent zone up to the boundary of the cavity.

In the case of the kinetic drift instability it is necessary to take into account the wave damping resulting from solar wind protons in the dispersion relation given by Eq. (19) and subsequently to make the following substitution:

$$\frac{1}{k^2 d_p^2} \rightarrow \frac{1}{k^2 d_p^2}\left(1 + i\sqrt{\frac{\pi}{2}}\frac{\omega}{k v_{Tp}}\right).$$

Since the excited waves have negative energy the wave damping leads to the instability but of a kinetic nature with the growth rate similar to that obtained by Drake *et al.*,[20] and given by

$$\gamma = {\rm Im}\,\epsilon = i\sqrt{\frac{2}{\pi}\frac{m_B}{m_p}\frac{n_e}{n_B}}\,\kappa r_{Lp}\frac{\omega_{\rm LH} k_y r^*}{\sqrt{1+\beta+k_y^2 r^{*2}}}, \qquad r_{Lp} = \sqrt{\frac{T_p}{m_p}}\frac{1}{\omega_{Hp}}. \quad (23)$$

In conclusion we find the following:

(1) The beam-driven lower-hybrid instability is possible only in the outer part of barium cloud, where $n_B < n_p m_p / m_B$.

(2) The main instability resulting from the barium beam, solar wind plasma interaction is the ion-acoustic instability; this is responsible for the wave excitation in the wide-frequency range from tens of hertz up to kilohertz.

(3) The current-driven lower-hybrid hydrodynamic drift instability is responsible for the low-frequency (of the order of tens of hertz or less) part of the spectrum in the whole of the turbulent zone.

V. NONLINEAR THEORY OF THE INSTABILITIES INVOLVED, ESTIMATION OF WAVE AMPLITUDES, AND ENERGETIC ELECTRON TAILS

As shown in Sec. III the main wave activity in AMPTE is connected with two instabilities, namely the current-driven lower-hybrid drift instability for the low-frequency range and the beam-driven ion-acoustic instability for the high-frequency part of the spectrum. Here we present estimates that give us the opportunity to compare the theoretical treatment with the observations. The detailed nonlinear theoretical analysis of the instabilities involved would be the subject of further work and here we restrict ourselves with a brief nonlinear description. For the lower-hybrid drift instability we will consider the most important nonlinear process to be the modulational instability. As a result of this instability, waves that have been initially excited in the plane perpendicular to the magnetic field acquire a component of the wave vector along the magnetic field, such that

$$k_\parallel \sim k\sqrt{m_e/m_B} \sim \omega_{\rm LH}/v_{Te}.$$

This results in the cascade of waves with high phase velocities along **B** to waves with phase velocities along **B** close to the electron thermal velocity and strong damping of the oscillations by electrons saturating the wave amplitude at the level of the threshold of the modulational instability.

The nonlinear mode coupling for lower-hybrid waves resulting in the modulational instability was first considered by Sotnikov *et al.*[19] for the case of oblique ($k_\parallel \neq 0$) waves.

Later on a more detailed treatment of mode coupling and parametric instability of lower-hybrid waves have been carried out in Ref. 20, but only for the case $k_{\parallel} = 0$. The nonlinear scenario proposed in these papers is the following. While reaching the amplitudes of the wave electric field potential

$$\phi \propto (T_i/e)\rho_e \kappa,$$

where ρ_e is the electron Larmor radius.

The instability saturates by transferring energy from unstable long wavelength modes to more easily damped short wavelength modes. The damping mechanism is resonant electron absorption[19] for the case $k_{\parallel} \neq 0$ or collisional absorption[21] for the case $k_{\parallel} = 0$. The treatment in Refs. 19 and 21 deals with finite β_i plasma with hot ions. Below we will consider the mode coupling for lower-hybrid waves under conditions more appropriate to the AMPTE releases when ions are cold and their pressure is much smaller than the magnetic pressure. Also, the velocity of barium ions in the solar wind frame will be taken into account.

A. Modulational instability of lower-hybrid waves

We will consider the case when the monochromatic lower-hybrid wave excited by the drift instability and polarized in the plane perpendicular to magnetic field acts as a pump for the parametric instability and generation of oblique ($k_{\parallel} \neq 0$) waves. In that case the wave potential can be written in the form

$$\phi = e^{i(\mathbf{k}_0\cdot\mathbf{r}_\perp - \omega_0 t)}\left[\phi_0 + \phi_+ e^{i(\mathbf{k}_\perp\cdot\mathbf{r}_\perp + k_\parallel z - \omega t)} + \phi_- e^{-i(\mathbf{k}_\perp\cdot\mathbf{r}_\perp + k_\parallel z - \omega t)}\right].$$

Here $\mathbf{k}_0$ and ω_0 are the wave vector and frequency of the pump wave, and

$$\omega_0 = -\mathbf{k}_0\cdot\mathbf{v}_{\rm sw} + \omega_{{\rm LH}B},$$

which follows from Eq. (19) in the limit $kr^* > 1$, i.e., when the proton term is unimportant. Because of the modulational instability, two satellites ϕ_+, ϕ_- with the wave vectors $\mathbf{k}_0 \pm \mathbf{k}$ are excited, resulting in the modulation of the pump wave amplitude. An analysis of the experimental data demonstrates that in the turbulent zone the magnetic field develops fine structure.[11] Such structure is due to the modulation of the magnetic field that has a characteristic space scale perpendicular to the magnetic field direction and very prolonged along the field lines and varying on a time scale much faster than the barium gyroperiod. We assume that this modulation is created by long wavelength magnetosonic waves that parametrically couple the pump wave to the lower-hybrid satellites $\phi_\pm$. Hence contrary to the usual treatment of the modulational instability of lower-hybrid waves in which the slow mode is coupled with density perturbations such as the ion-acoustic mode we will consider the case when the slowly varying low-frequency mode is associated with the magnetosonic mode with the instability, resulting in magnetic field modulations. In accordance with experimental data we will also assume that for these perturbations $k_\parallel \ll k_\perp$. Using the drift approximation for electrons $|d/dt| \ll \omega_{He}$ and supposing that the ions are unmagnetized for the lower-hybrid mode, $|d/dt| \gg \omega_{HB}$, it is possible to obtain the following equation for the electric field of lower-hybrid waves in the case of quasineutrality $n_e' = n_i'$:

$$\left(\frac{\partial}{\partial t} + u_{ey}\frac{\partial}{\partial y}\right)^2\left[1 + \frac{1}{\omega_{{\rm LH}B}^2}\left(\frac{\partial}{\partial t} - \mathbf{v}_{\rm sw}\cdot\nabla\right)^2\right]\nabla\cdot\mathbf{E}_\perp' + \frac{m_B}{m_e}\left(\frac{\partial}{\partial t} - \mathbf{v}_{\rm sw}\cdot\nabla\right)^2\frac{\partial E_z'}{\partial z} - \left(\frac{\partial}{\partial t} + u_{ey}\frac{\partial}{\partial y}\right)\times\left(\frac{\partial}{\partial t} - \mathbf{v}_{\rm sw}\cdot\nabla\right)^2\kappa\frac{E_y'}{\omega_{HB}} - \frac{1}{\omega_{HB}}\left(\frac{\partial}{\partial t} + u_{ey}\frac{\partial}{\partial y}\right)\times\left(\frac{\partial}{\partial t} - \mathbf{v}_{\rm sw}\cdot\nabla\right)^2\left(E_x'\frac{\partial}{\partial y} - E_y'\frac{\partial}{\partial x}\right)\left(\frac{\delta H}{H_0} - \frac{\delta n}{n_0}\right) = 0. \quad (24)$$

Here the main nonlinear term is the $\nabla\cdot(n_e\mathbf{v}_e)$ nonlinearity in the electron continuity equation, δn and δH are the density and magnetic field modulations resulting from the slow magnetosonic wave. Neglecting in Eq. (24) the term proportional to κ that is responsible for the drift instability of the pump wave it is possible to obtain the following relation for the wave potential of the two lower-hybrid satellites:

$$\phi_+ = -\frac{i\omega_{{\rm LH}B}^2[\mathbf{k}_0\times\mathbf{k}]_z}{\omega_{HB}(\omega - k_y u_{ey})}\frac{\phi_0}{(\mathbf{k}_0+\mathbf{k})^2}\left(\frac{\delta H}{H_0} - \frac{\delta n}{n_0}\right)\times\frac{1}{\delta_+ - 2[(\omega + \mathbf{k}\cdot\mathbf{v}_{\rm sw})/\omega_{{\rm LH}B}]}, \quad (25a)$$

$$\phi_- = -\frac{i\omega_{{\rm LH}B}^2[\mathbf{k}_0\times\mathbf{k}]_z}{\omega_{HB}(\omega - k_y u_{ey})}\frac{\phi_0}{(\mathbf{k}_0-\mathbf{k})^2}\left(\frac{\delta H^*}{H_0} - \frac{\delta n^*}{n_0}\right)\times\frac{1}{\delta_- + 2[(\omega + \mathbf{k}\cdot\mathbf{v}_{\rm sw})/\omega_{{\rm LH}B}]}, \quad (25b)$$

where we have used the fact that for pump wave resonance $\omega_0 = k_{y0}u_{ey}$ and

$$\delta\pm = \frac{m_B}{m_e}\frac{k_\parallel^2}{(\mathbf{k}_0 \pm \mathbf{k})^2}\frac{\omega_{{\rm LH}B}^2}{(\omega - k_y u_{ey})^2}$$

is the detuning between the frequency of the pump wave and the satellites. For simplicity we will suppose that the lower-frequency mode is mainly inhomogeneous along the direction of the current ($k_y \neq 0$) and very prolonged along the magnetic field ($k_\parallel \ll k_y$). The equation connecting the density and magnetic field fluctuations is the usual one for magnetosonic waves:

$$\left(\frac{\partial}{\partial t} - \mathbf{v}_{\rm sw}\cdot\nabla\right)^2\delta n = v_{{\rm A}B}^2\frac{n_0}{H_0}\frac{\partial^2\delta H}{\partial y^2}; \quad (26)$$

$v_{{\rm A}B} = \sqrt{H/4\pi n_B m_B}$ is the Alfvén velocity for barium ions.

While obtaining the equation for δH it is necessary to use the following relation between the electron and ion velocities in the direction of inhomogeneity:

$$\left(\frac{\partial}{\partial t} + u_{ey}\frac{\partial}{\partial y}\right)\delta v_{iy} = \left(\frac{\partial}{\partial t} - \mathbf{v}_{\rm sw}\cdot\nabla\right)\delta v_{ey}. \quad (27)$$

This relation is the consequence of quasineutrality of the slow magnetosonic wave, i.e., $\delta n_e = \delta n_i$. Then the equation for δH has the following form:

$$\left(\frac{\partial}{\partial t}+u_{ey}\frac{\partial}{\partial y}\right)\left[\frac{\partial^2\delta H}{\partial y^2}-\frac{1}{v_{AB}^2}\left(\frac{\partial}{\partial t}-\mathbf{v}_{sw}\cdot\nabla\right)^2\delta H\right]$$
$$=-\frac{4\pi e n_0 c}{H_0^2\omega_{LHB}^2}\left(\frac{\partial}{\partial t}-\mathbf{v}_{sw}\cdot\nabla\right)^2$$
$$\times\left(\frac{\partial\phi'}{\partial x}\frac{\partial}{\partial y}-\frac{\partial\phi'}{\partial y}\frac{\partial}{\partial x}\right)\frac{\partial^2\phi'}{\partial y^2}. \quad (28)$$

The relation for the magnetic field modulation resulting from the coupling between the pump lower-hybrid wave and the satellites follows from Eq. (28), and is given by

$$\delta H=\frac{4\pi e n_0 c}{H_0^2}\frac{i}{\omega_{LHB}^2}[\mathbf{k}_0\times\mathbf{k}]_z\frac{(\omega+\mathbf{k}\cdot\mathbf{v}_{sw})^2}{(\omega-k_y u_{ey})}$$
$$\times\frac{1}{1-(\omega+\mathbf{k}\cdot\mathbf{v}_{sw})^2/k_y^2v_{AB}^2}\bigg((\phi_+\phi_0^*-\phi_-^*\phi_0)$$
$$+\frac{2k_{0y}}{k_y}(\phi_+\phi_0^*+\phi_-^*\phi_0)\bigg). \quad (29)$$

We restrict ourselves to the consideration of the case when, as the result of the modulation, sufficiently long wavelength magnetic structures are created, i.e., $k_y \ll k_{0y}$. In that case the dispersion relation has the following form:

$$1=8\frac{\omega+\mathbf{k}\cdot\mathbf{v}_{sw}}{\omega_{LHB}}\frac{k_{0y}}{k_y}[\mathbf{k}_0\times\mathbf{k}]_z^2\frac{c^2|\phi_0|^2}{H_0^2}$$
$$\times\frac{1}{(\omega-k_yu_{ey})^2}\cdot\frac{1}{\delta_0^2-4(\omega-k_yu_{ey})^2/\omega_{LHB}^2}. \quad (30)$$

It follows from the dispersion relation that the maximum growth rate corresponds to the resonant case $k_yu_{ey}+\mathbf{k}\cdot\mathbf{v}_{sw}=0$.

For a sufficiently strong pump when the condition

$$\frac{k_{0y}}{k_y}\frac{c^2|E_0|^2}{H_0^2}>\frac{\omega_{LHB}^2}{k^2}\frac{m_B}{m_e}\frac{k_\parallel^2}{k^2} \quad (31)$$

is fulfilled, the maximum growth rate obtained from Eq. (30) is equal to

$$\gamma_{max}=\frac{\sqrt{3}}{2^{2/3}}\left(\omega_{LHB}\frac{k_{0y}}{k_y}[\mathbf{k}_0\times\mathbf{k}]_z^2\frac{c^2|\phi_0|^2}{H_0^2}\right)^{1/3}. \quad (32)$$

The magnetic structures arising as the result of modulational instability are convected with the solar wind and, in order to give the instability sufficient time to grow, the following condition must be fulfilled: $\gamma_{max}L/v_{sw}>10$, where L is the size of the cloud.

For $L\sim100$ km, $v_{sw}\sim3.10^7$ cm/sec we need a value of $\gamma_{max}\sim30$ Hz and it follows from Eq. (32) that for $H=3.10^{-3}$ G, $u_{ey}\sim10^6$ cm/sec, $k_0\simeq\omega_{LHB}/u_e\sim10^{-4}$ cm^{-1}, and $k/k_0\sim\frac{1}{3}$, the typical lower-hybrid wave amplitude needed for the development of the instability is $E_0\sim(3\text{–}5)$ mV/m, which is close to the observed values. The modulational instability not only results in the creation of magnetic structures slowly varying with time but also in the appearance in the lower-hybrid wave spectrum waves with $k_\parallel\neq0$. The waves with $k_\parallel\sim(1/3v_{Te})\omega_{LHB}$ or $k_\parallel/k_0\sim u_{ey}/3v_{Te}\sim\frac{1}{3}\times10^{-2}$ will be efficiently absorbed by resonant electrons, this absorption will prevent growth of the pump amplitude causing saturation of the pump wave. In the stationary situation the rate of flow of energy into the current-driven lower-hybrid transverse mode, i.e., $\gamma_0|E_0|^2$, must be balanced by the rate of energy flow into the oblique mode by the modulational instability, i.e., $\gamma_{max}|E|^2$. The fact that the growth rate of the current-driven lower-hybrid instability γ_0 is of the order of that for the modulational instability γ_{max} results in comparable amplitudes of transverse and oblique modes.

For the beam-driven lower-hybrid instability the nonlinear saturation mechanism could also be connected with the trapping of the proton beam in a potential well created by the excited wave (see Refs. 5 and 20). In this case the amplitude of the wave potential is to be obtained from the trapping condition

$$\sqrt{e\phi/m_p}\propto\gamma/k\propto\alpha^{1/3}v_{sw}.$$

It is easy to see that this condition corresponds to the following wave energy for lower-hybrid oscillations:

$$W_E=(E^2/4\pi)(\omega_{pe}^2/\omega_{He}^2)\simeq\alpha^{1/3}n_pm_pv_{sw}^2. \quad (33)$$

It is also necessary to mention the saturation mechanism for lower-hybrid waves based on wave damping as a result of stochastization of electron trajectories in the plane perpendicular to the magnetic field.[22] The wave amplitudes saturate resulting from stochastization at values given by[22]

$$k_y^2c^2|E_0|^2/H_0^2\sim\omega_{He}^2,$$

which is sufficiently (at least by factor $\sqrt{m_b/m_e}$) larger than those obtained from Eq. (31) corresponding to the modulational instability. Hence the latter, i.e., the modulational instability seems to be the dominant process for the stabilization of the lower-hybrid waves.

B. Electron heating

Since the parametrically excited lower-hybrid waves have a finite longitudinal electric field $E_\parallel$, they can stochastically accelerate electrons that fall into Čerenkov resonance ($\omega=k_\parallel v_\parallel$) with them. The detailed calculation of this effect will be presented in a future paper. Here we will give an estimate for the energy gained by these resonant electrons based on the quasilinear diffusion equation

$$v_e\frac{\partial f}{\partial z}=\frac{\partial}{\partial v_\parallel}D_\parallel\frac{\partial f_e}{\partial v_\parallel}, \quad (34)$$

where f_e is the electron distribution function and $D_\parallel$ is the diffusion coefficient in the parallel direction, given by

$$D_\parallel=\frac{e^2}{m_e^2}|E_k|^2\left(\frac{k_\parallel}{k}\right)^2\frac{1}{v_\parallel}\quad\left(k_\parallel=\frac{\omega_{LHB}}{v_\parallel}\right).$$

It is easy to obtain the following scaling law for the characteristic energy gained by resonant electrons while flowing through the turbulent zone:

$$\mathscr{E}_e\simeq\left(\frac{\omega_{pe}^2}{\omega_{He}^2}\frac{|E|^2}{4\pi n_e}m_ev_{Ae}^2\omega_{LHB}L\sqrt{m_e}\right)^{2/5}. \quad (35)$$

Using the following values, for the magnetic field, plasma electron density, and turbulent wave field of $H\sim100$ nT, $n_e\simeq10^2$, and $E\sim10$ mV/m, and distance L, representing the size of the turbulent zone of order 300 km we have from Eq. (35) an estimate of the typical energies of the accelerated electrons to be ~ 100 eV, which agrees very well with the observations.[7] We can also estimate the fraction of electrons accelerated in the tail by equating the rate at which energy is cascading into larger parallel wave numbers $k_\parallel$ as a result of the modulational instability to the Landau damping rate of these waves by electrons. The Landau damping rate of waves on electrons is given by

$$\gamma_e \simeq \omega_{\rm LH}(n_{\rm TAIL}/n_0)(\omega_{\rm LH}^2/k_\parallel^2 v_{Te}^2). \tag{36}$$

This results in $n_{\rm TAIL}/n_0$ of order $\gamma_{\max}/\omega_{\rm LH}$, where $\gamma_{\max}$ is modulational growth rate, defined by Eq. (32), resulting in a $n_{\rm TAIL}/n_0$ of about 1%–10%.

C. Ion-acoustic waves

The instability that drives the ion-acoustic waves unstable is of the dissipative type and is caused by the cold beam of barium ions flowing through the solar wind plasma. Here we present a qualitative description of the dynamics for the case $\alpha_0 < 1$. The energy source of the instability is the relative flow between the solar wind and the released ions. For the situation we consider, i.e., when $n_p m_p \ll n_B m_B$, no significant change occurs in the barium ion velocity distribution. The velocity spread of the barium ions caused by the instability cannot exceed the value $\Delta v/v \leqslant (n_p/n_B) \times (m_p/m_B) < (10^{-3}–10^{-4})$. In this case the main quasilinear effect that can stabilize the wave growth is the formation of a plateau on the resonant proton distribution function. The formation of a plateau switches off the dissipation mechanism. Since for oblique ion-acoustic waves the whole bulk of thermal protons is in resonance with the waves and the energy gained by these particles resulting from plateau formation is

$$W_p \simeq n_p T_p. \tag{37}$$

It follows from the condition of instability, i.e., Eq. (17) that the excited waves are propagating almost perpendicular to the direction of relative motion,

$$\theta = \frac{\mathbf{k}\cdot\mathbf{v}_{\rm sw}}{|k\cdot v_{\rm sw}|} \simeq \alpha_0^{1/2}\frac{\omega_{\rm pp} d_*}{v_{\rm sw}} \simeq \sqrt{\frac{T_e}{m_B v_{\rm sw}^2}} \ll 1.$$

This results in predominantly elastic scattering of the barium ion beam by waves. The change of longitudinal energy of the beam in such an elastic scattering process is equal to

$$\Delta W_B \simeq n_B m_B v_{\rm sw}\,\Delta v_B \simeq \eta n_p m_p v_{\rm sw}^2, \tag{38}$$

which also corresponds to momentum balance between the barium ions and the solar wind, where η is the fraction of solar wind momentum lost in this interaction.

Only small amounts of this energy ΔW_B of order of θ is going into the waves; hence

$$W_E \simeq \sqrt{T_e/m_B v_{\rm sw}^2}\,\Delta W_B \tag{39}$$

is the energy in the waves.

Since

$$\frac{\omega}{k v_{Tp}} \simeq \alpha_0^{1/2}\omega_{\rm pp}\frac{d_*}{\mathbf{v}_{Tp}} \simeq \sqrt{\frac{T_e}{T_p}\frac{m_p}{m_B}} \ll 1,$$

the interaction of these waves with the thermal protons is also almost an elastic one. The energy in the waves W_E is small in comparison with the energy gained by thermal protons in the process of their quasilinear diffusion just by the factor $\omega/k\mathbf{v}_{Tp}$, i.e.,

$$W_p \simeq W_E\sqrt{(T_p/T_e)(m_B/m_p)}. \tag{40}$$

Using the value of W_p from Eq. (37) it is easy to obtain the final result for thermal proton heating:

$$T_p \simeq \eta^2 m_p v_{\rm sw}^2. \tag{41}$$

The conclusion about significant proton heating at the nonlinear stage of the ion-acoustic instability agrees reasonably well with the observational data observed in the releases[7] as well as with the results of numerical simulation.[23]

If we assume that the 70% of the solar wind energy lost is due to the instability under consideration, then $\eta \simeq \frac{2}{3}$, and it follows from Eq. (41) that $T_p \simeq 0.65$ keV, which is somewhat larger than the observed value of $T_p \simeq 0.4$ keV. The latter corresponds to $\eta \simeq 0.5$ and may serve as an indication that the deceleration of the solar wind flow in the compressed region by the dc electric field, resulting from gradients in the magnetic field, is also important. The wave energy obtained from the Eqs. (40) and (41) given by

$$W_E = |E|^2/4\pi \simeq \eta^2\sqrt{m_p T_e/m_B T_p}\, n_p m_p v_{\rm sw}^2 \tag{42}$$

is small in comparison with the energy lost by the solar wind flow by the factor $\sqrt{m_p/m_B}$. Nevertheless, the wave electric field calculated from Eq. (42) for the typical solar wind parameters $n_p \approx 5\ {\rm cm}^{-3}$, $\mathbf{v}_{\rm sw} \approx 270$ km/sec, and $\eta \simeq 0.5$ is quite large and corresponds to spectral energy density in the kilohertz range $\sqrt{E_f^2}$ of about 10–15 mV/m$\sqrt{\rm Hz}$, which agrees with the observed values.

VI. CONCLUSIONS

In this paper we have proposed possible mechanisms for the generation of the broadband wave turbulence observed in the AMPTE release experiments. The mechanisms are based on two types of instabilities that are possible in the cloud of the expanding barium ions interacting with the solar wind. One is the barium beam-driven instability resulting in the excitation of long wavelength ($\lambda \geqslant 1$ km) lower-hybrid waves, as well as short wavelength ($\lambda \leqslant 10–100$ m) ion-acoustic oscillations. Another is the lower-hybrid drift instability driven by currents associated with the large magnetic field gradients in the turbulent zone. The frequencies of the excited waves change from about 5–10 Hz in the lower-hybrid range up to 1 kHz for ion-acoustic waves. The analysis of the instabilities presented shows the following.

(1) The modulational instability of lower-hybrid waves could result in magnetic field modulation aligned along the magnetic field direction, which are observed both in the experiment[11] and in the simulations.[24]

(2) Excitation resulting from the modulational instability of oblique lower-hybrid waves with $k_\parallel \neq 0$, producing a cascade in wave number to larger values, leads to the absorption of the wave energy by resonant electrons producing en-

ergetic field aligned electron tails with energies greater than 100 eV, which also agrees with the observations.[7]

(3) The nonlinear estimates derived from the saturation of the wave spectrum by the modulational instability for lower-hybrid waves of the electric field amplitude give values of E in the range 5–15 mV/m. For the ion-acoustic waves that heat the ions the estimated electric field amplitude is $E \sim 300$ mV/m, which is in very good agreement with the experiment.[8]

(4) The wave energy is not enough for significant "pickup" of the barium ions by the solar wind some of the heating of the barium ions may be mainly due to the lower-hybrid drift instability. This is the dissipative instability of the negative energy waves resonant with the thermal ions, with the growth of the wave amplitude resulting in energy gain of the thermal particles. The same physics applies in respect to the ion-acoustic instability, which is also a negative energy instability and could serve as the reason for the significant heating of the thermal solar wind protons.

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